§2.5 Independence

Def: Two events A & B are *independent* if

$$P(A \mid B) = P(A)$$

(equivalent to $P(B \mid A) = P(B)$)

(equivalent to $P(A \cap B) = P(A) \cdot P(B)$)

The idea is that independent events are "unrelated"

$\Rightarrow$ Knowing one event happened gives no extra information about whether other event occurred.

Example: Roll two dice.

Events about die #1 are independent from events about die #2.

$\Rightarrow$ Ex: $P\left(\begin{smallmatrix}\text{die \#1}\\ \text{is even}\end{smallmatrix} \;\middle|\; \begin{smallmatrix}\text{die \#2}\\ \text{is 3}\end{smallmatrix}\right) = P\left(\begin{smallmatrix}\text{die \#1}\\ \text{is even}\end{smallmatrix}\right)$

$\Rightarrow$ Ex: $P\left(\begin{smallmatrix}\text{die \#2}\\ \text{is 3}\end{smallmatrix} \;\middle|\; \begin{smallmatrix}\text{die \#1}\\ \text{is 4}\end{smallmatrix}\right) = P\left(\begin{smallmatrix}\text{die \#2}\\ \text{is 3}\end{smallmatrix}\right)$

Example: Pick a card from a deck.

Events about number are independent from events about suit.

$\Rightarrow$ Ex: $P\left(\begin{smallmatrix}\text{card is}\\ \text{king}\end{smallmatrix} \;\middle|\; \begin{smallmatrix}\text{suit is}\\ \heartsuit\end{smallmatrix}\right) = P\left(\begin{smallmatrix}\text{card is}\\ \text{king}\end{smallmatrix}\right)$

$\Rightarrow$ Ex: $P\left(\begin{smallmatrix}\text{suit is}\\ \text{Red}\end{smallmatrix} \;\middle|\; \begin{smallmatrix}\text{card is}\\ \text{face}\end{smallmatrix}\right) = P\left(\begin{smallmatrix}\text{suit is}\\ \text{Red}\end{smallmatrix}\right)$

The standard example for independent events is when unrelated experiments are performed in sequence as in first example (roll two dice), but you can get independence in other setups, too

Example: Roll one die

Event $A = \{\text{roll} \le 4\}$

Event $B = \{\text{roll even}\}$

$P(A) = 4/6 \qquad P(A \cap B) = 2/6$

$P(B) = 3/6$

$$\Rightarrow P(A \mid B) = \frac{2/6}{3/6} = 2/3 = P(A) \quad \text{Independent!}$$

$$\left(\text{Also } P(B \mid A) = \frac{2/6}{4/6} = 2/4 = P(B)\right)$$

Note: The probability table for independent events is merely a table of products:

Prob.	A	B	Total
B	$P(A)P(B)$	$[1-P(A)]P(B)$	$P(B)$
not B	$P(A)[1-P(B)]$	$[1-P(A)][1-P(B)]$	$1-P(B)$
Total	$P(A)$	$1-P(A)$	1

Example Probability table for experiment "Roll two dice"

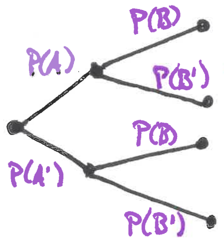
Event #1: Die #1 is ≤ 4

Event #2: Die #2 is 2

Prob	Die #1 ≤ 4	Die #1 ≥ 5	Total
Die #2 $= 2$	$4/36$	$2/36$	$1/6$
Die #2 $\neq 2$	$20/36$	$10/36$	$5/6$
Total	$4/6$	$2/6$	1

Trees for independent events are also simple

→ all subtrees at each level are identical:



Identical subtrees because

$P(B \mid A) = P(B)$

and

$P(B \mid A') = P(B)$ also!